

# Heavy dibaryons

S.M. Gerasyuta\* and E.E. Matskevich†

*Department of Theoretical Physics, St. Petersburg State University, 198904, St. Petersburg, Russia and  
Department of Physics, LTA, 194021, St. Petersburg, Russia*

The relativistic six-quark equations are found in the framework of the dispersion relation technique. The approximate solutions of these equations using the method based on the extraction of leading singularities of the heavy hexaquark amplitudes are obtained. The relativistic six-quark amplitudes of dibaryons including the light quarks  $u$ ,  $d$  and heavy quarks  $c$ ,  $b$  are calculated. The poles of these amplitudes determine the masses of charmed and bottom dibaryons with the isospins  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{5}{2}$ .

PACS numbers: 11.55.Fv, 12.39.Ki, 12.39.Mk, 12.40.Yx.

## I. INTRODUCTION.

Hadron spectroscopy has always played an important role in the revealing mechanisms underlying the dynamic of strong interactions.

The heavy hadron containing a single heavy quark is particularly interesting. The light degrees of freedom (quarks and gluons) circle around the nearby static heavy quark. Such a system behaves as the QCD analog of familiar hydrogen bound by the electromagnetic interaction.

The heavy quark expansion provides a systematic tool for heavy hadrons. When the heavy quark mass  $m_Q \rightarrow \infty$ , the angular momentum of the light degree of freedom is a good quantum number. Therefore, heavy hadrons form doublets. For example,  $\Omega_b$  and  $\Omega_b^*$  will be degenerate in the heavy quark limit. Their mass splitting is caused by the chromomagnetic interaction at the order  $O(1/m_Q)$ , which can be taken into account systematically in the framework of heavy quark effective field theory (HQET) [1–3].

In 1977, Jaffe [4] studied the color-magnetic interaction of the one-gluon-exchange potential in the multiquark system and found that the most attractive channel is the flavor singlet with quark content  $u^2 d^2 s^2$ . The same symmetry analysis of the chiral boson exchange potential leads to the similar result [5].

The  $H$ -particle,  $N\Omega$ -state and di- $\Omega$  may be strong interaction stable. Up to now, these three interesting candidates of dibaryons are still not found or confirmed by experiments. It seems that one should go beyond these candidates and should search the possible candidates in a wider region, especially the systems with heavy quarks, in terms of a more reliable model.

There were a number of theoretical predictions by using various models: the quark cluster model [6, 7], the quark-delocation model [8, 9], the chiral  $SU(3)$  quark model [10], the flavor  $SU(3)$  skyrmion model [11]. Lomon predicted a deuteronlike dibaryon resonance using R-matrix theory [12]. By employing the chiral  $SU(3)$  quark model Zhang and Yu studied  $\Omega\Omega$  and  $\Sigma\Omega$  states [13, 14].

In a series of papers [15–19] a method has been developed which is convenient for analyzing relativistic three-hadron systems. The physics of the three-hadron system can be described by means of a pair interaction between the particles. There are three isobar channels, each of which consists of a two-particle isobar and the third particle. The presence of the isobar representation together with the condition of unitarity in the pair energies and of analyticity leads to a system of integral equations in a single variable. Their solution makes it possible to describe the interaction of the produced particles in three-hadron systems.

In Ref. [20] a representation of the Faddeev equation in the form of a dispersion relation in the pair energy in the two interacting particles was used. This was found to be convenient in order to obtain an approximate solution of the Faddeev equation by a method based on extraction of the leading singularities of the amplitude. With a rather crude approximation of the low-energy  $NN$  interaction a relatively good description of the form factor of tritium (helium-3) at low  $q^2$  was obtained.

In our papers [21–23] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting quarks. The mass spectrum of  $S$ -wave baryons including  $u$ ,  $d$ ,

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\*Electronic address: gerasyuta@SG6488.spb.edu

†Electronic address: matskev@pobox.spbu.ru

$s$  quarks was calculated by a method based on isolating the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered such approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions of the subenergy variables (as compared with the singular part of the amplitude) in the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In the present paper the relativistic six-quark equations are found in the framework of coupled-channel formalism. We use only planar diagrams; the other diagrams due to the rules of  $1/N_c$  expansion [24–26] are neglected.

The six-quark amplitudes of dibaryons are calculated. The poles of these amplitudes determine the masses of dibaryons. We calculated the contribution of six-quark subamplitudes to the hexaquark amplitudes. In Sec. II, we briefly discuss the relativistic Faddeev approach. The relativistic three-quark equations are constructed in the form of the dispersion relation over the two-body subenergy. The approximate solution of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. We calculated the mass spectrum of  $S$ -wave bottom baryons with  $J^P = \frac{1}{2}^+, \frac{3}{2}^+$  (Tables I, II). In Sec. III, the six-quark amplitudes of hexaquarks are constructed. The dynamical mixing between the subamplitudes of dibaryons are considered. The relativistic six-quark equations are constructed in the form of the dispersion relation over the two-body subenergy. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. Sec. IV is devoted to the calculation results for the dibaryon mass spectra and the contributions of subamplitudes to the hexaquark amplitude (Tables IV, V, VI, VII). In conclusion, the status of the considered model is discussed.

## II. BRIEF INTRODUCTION OF RELATIVISTIC FADDEEV EQUATIONS.

In our papers, [21–23, 27, 28] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting particles. The mass spectra of  $S$ -wave baryons including  $u, d, s, c$  quarks were calculated by a method based on isolating of the leading singularities in the amplitude.

We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, the weaker ones being neglected. If we considered such an approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions at the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In the paper [27] the relativistic three-particle amplitudes in the coupled-channels formalism are considered. We take into account the  $u, d, s, c, b$  quarks and construct the flavor-spin functions for the 35 baryons with the spin-parity  $J^P = \frac{1}{2}^+$  and  $J^P = \frac{3}{2}^+$ :

$J^P = \frac{1}{2}^+$	$J^P = \frac{3}{2}^+$
$\Sigma_b$ $uub, udb, ddb$	$\Sigma_b$ $uub, udb, ddb$
$\Lambda_b$ $udb$	$\Xi_{sb}$ $usb, dsb$
$\Xi_{sb}^A$ $usb, dsb$	$\Omega_{ssb}$ $ssb$
$\Xi_{sb}^S$ $usb, dsb$	$\Xi_{cb}$ $ucb, dcb$
$\Omega_{ssb}$ $ssb$	$\Omega_{scb}$ $scb$
$\Xi_{cb}^A$ $ucb, dcb$	$\Omega_{ccb}$ $ccb$
$\Xi_{cb}^S$ $ucb, dcb$	$\Xi_{bb}$ $ubb, dbb$
$\Lambda_{scb}^A$ $scb$	$\Omega_{sbb}$ $sbb$
$\Lambda_{scb}^S$ $scb$	$\Omega_{cbb}$ $cbb$
$\Omega_{ccb}$ $ccb$	$\Omega_{bbb}$ $bbb$
$\Xi_{bb}$ $ubb, dbb$	
$\Omega_{sbb}$ $sbb$	
$\Omega_{cbb}$ $cbb$	

In the paper [28], the relativistic equations were obtained and the mass spectrum of  $S$ -wave charmed baryons was calculated.

In the present paper, we will be able to use the similar method. In this case, we consider 35 baryons with the

spin-parity  $J^P = \frac{1}{2}^+$  and  $J^P = \frac{3}{2}^+$ , which include one, two and three bottom quarks. We have considered the 23 baryons with different masses (Tables I, II).

We calculate the masses of the bottom baryons in a relativistic approach using the dispersion relation technique. The relativistic three-quark integral equations are constructed in the form of the dispersion relations over the two-body subenergy.

We use the graphical equations for the functions  $A_J(s, s_{ik})$ . In order to represent the amplitude  $A_J(s, s_{ik})$  in the form of dispersion relations, it is necessary to define the amplitudes of quark-quark interaction  $a_J(s_{ik})$ . The pair amplitudes  $qq \rightarrow qq$  are calculated in the framework of the dispersion  $N/D$  method with the input four-fermion interaction with quantum numbers of the gluon. We use results of our relativistic quark model [29] and write down the pair quark amplitudes in the following form:

$$a_J(s_{ik}) = \frac{G_J^2(s_{ik})}{1 - B_J(s_{ik})}, \quad (1)$$

$$B_J(s_{ik}) = \int_{(m_i+m_k)^2}^{\Lambda_J(i,k)} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik}) G_J^2(s'_{ik})}{s'_{ik} - s_{ik}}, \quad (2)$$

$$\begin{aligned} \rho_J(s_{ik}) &= \frac{(m_i + m_k)^2}{4\pi} \left( \alpha_J \frac{s_{ik}}{(m_i + m_k)^2} + \beta_J + \frac{\delta_J}{s_{ik}} \right) \\ &\times \frac{\sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)}}{s_{ik}}. \end{aligned} \quad (3)$$

Here  $G_J$  is the vertex function of a diquark, which can be expressed in terms of the  $N$ -function of the bootstrap  $N/D$  method as  $G_J = \sqrt{N_J}$ ,  $B_J(s_{ik})$  is the Chew-Mandelstam function [30], and  $\rho_J(s_{ik})$  is the phase space for a diquark.  $s_{ik}$  is the pair energy squared of diquark, the index  $J^P$  determines the spin-parity of diquark. The coefficients of Chew-Mandelstam function  $\alpha_J$ ,  $\beta_J$  and  $\delta_J$  in Table III are given.  $\Lambda_J(i, k)$  is the pair energy cutoff. In the case under discussion the interacting pairs of quarks do not form bound states. Therefore, the integration in the dispersion integral (2) is carried out from  $(m_i + m_k)^2$  to  $\Lambda_J(i, k)$  ( $i, k = 1, 2, 3$ ). Including all possible rescatterings of each pair of quarks and grouping the terms according to the final states of the particles, we obtained the coupled systems of integral equations. For instance, for the  $\Sigma_b^+$  with  $J^P = \frac{1}{2}^+$  the wave function is  $\varphi_{\Sigma_b^+} = \sqrt{\frac{2}{3}}\{u \uparrow d \uparrow b \downarrow\} - \sqrt{\frac{1}{6}}\{u \uparrow d \downarrow b \uparrow\} - \sqrt{\frac{1}{6}}\{u \downarrow d \uparrow b \uparrow\}$ . Then the coupled system of equations has the following form:

$$\left\{ \begin{aligned} A_1(s, s_{12}) &= \lambda b_1(s_{12}) L_1(s_{12}) + K_1(s_{12}) \left[ \frac{1}{4} A_{1^b}(s, s_{13}) + \frac{3}{4} A_{0^b}(s, s_{13}) + \right. \\ &\quad \left. + \frac{1}{4} A_{1^b}(s, s_{23}) + \frac{3}{4} A_{0^b}(s, s_{23}) \right] \\ A_{1^b}(s, s_{13}) &= \lambda b_{1^b}(s_{13}) L_{1^b}(s_{13}) + K_{1^b}(s_{13}) \left[ \frac{1}{2} A_1(s, s_{12}) - \frac{1}{4} A_{1^b}(s, s_{12}) + \right. \\ &\quad \left. + \frac{3}{4} A_{0^b}(s, s_{12}) + \frac{1}{2} A_1(s, s_{23}) - \frac{1}{4} A_{1^b}(s, s_{23}) + \frac{3}{4} A_{0^b}(s, s_{23}) \right] \\ A_{0^b}(s, s_{23}) &= \lambda b_{0^b}(s_{23}) L_{0^b}(s_{23}) + K_{0^b}(s_{23}) \left[ \frac{1}{2} A_1(s, s_{12}) + \frac{1}{4} A_{1^b}(s, s_{12}) + \right. \\ &\quad \left. + \frac{1}{4} A_{0^b}(s, s_{12}) + \frac{1}{2} A_1(s, s_{13}) + \frac{1}{4} A_{1^b}(s, s_{13}) + \frac{1}{4} A_{0^b}(s, s_{13}) \right]. \end{aligned} \right. \quad (4)$$

Here the function  $L_J(s_{ik})$  has the form

$$L_J(s_{ik}) = \frac{G_J(s_{ik})}{1 - B_J(s_{ik})}. \quad (5)$$

The integral operator  $K_J(s_{ik})$  is

$$K_J(s_{ik}) = L_J(s_{ik}) \int_{(m_i+m_k)^2}^{\Lambda_J(ik)} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik})G_J(s'_{ik})}{s'_{ik} - s_{ik}} \int_{-1}^1 \frac{dz}{2}. \quad (6)$$

The function  $b_J(s_{ik})$  is the truncated function of Chew-Mandelstam:

$$b_J(s_{ik}) = \int_{(m_i+m_k)^2}^{\infty} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik})G_J(s'_{ik})}{s'_{ik} - s_{ik}}, \quad (7)$$

$z$  is the cosine of the angle between the relative momentum of particles  $i$  and  $k$  in the intermediate state and the momentum of particle  $j$  in the final state, taken in the c.m. of the particles  $i$  and  $k$ . Let some current produces three quarks with the vertex constant  $\lambda$ . This constant do not affect to the spectra mass of bottom baryons. By analogy with the  $\Sigma_b^+$  state, we obtain the rescattering amplitudes of the three various quarks for the all bottom states.

Let us extract two-particle singularities in  $A_J(s, s_{ik})$ :

$$A_J(s, s_{ik}) = \frac{\alpha_J(s, s_{ik})b_J(s_{ik})G_J(s_{ik})}{1 - B_J(s_{ik})}, \quad (8)$$

$\alpha_J(s, s_{ik})$  is the reduced amplitude. Accordingly to this, all integral equations can be rewritten using the reduced amplitudes. The function  $\alpha_J(s, s_{ik})$  is the smooth function of  $s_{ik}$  as compared with the singular part of the amplitude. We do not extract the three-body singularities, because they are weaker than the two-particle singularities. For instance, one considers the first equation of system for the  $\Sigma_b^+$  with  $JP = \frac{1}{2}^+$ :

$$\begin{aligned} \alpha_1(s, s_{12}) = & \lambda + \frac{1}{b_1(s_{12})} \int_{(m_1+m_2)^2}^{\Lambda_1(1,2)} \frac{ds'_{12}}{\pi} \frac{\rho_1(s'_{12})G_1(s'_{12})}{s'_{12} - s_{12}} \\ & \times \int_{-1}^1 \frac{dz}{2} \left( \frac{G_{1^b}(s'_{13})b_{1^b}(s'_{13})}{1 - B_{1^b}(s'_{13})} \frac{1}{2} \alpha_{1^b}(s, s'_{13}) + \frac{G_{0^b}(s'_{13})b_{0^b}(s'_{13})}{1 - B_{0^b}(s'_{13})} \frac{3}{2} \alpha_{0^b}(s, s'_{13}) \right). \end{aligned} \quad (9)$$

The connection between  $s'_{12}$  and  $s'_{13}$  is [21]:

$$\begin{aligned} s'_{13} = & m_1^2 + m_3^2 - \frac{(s'_{12} + m_3^2 - s)(s'_{12} + m_1^2 - m_2^2)}{2s'_{12}} \\ & \pm \frac{z}{2s'_{12}} \sqrt{(s'_{12} - (m_1 + m_2)^2)(s'_{12} - (m_1 - m_2)^2)(s'_{12} - (\sqrt{s} + m_3)^2)(s'_{12} - (\sqrt{s} - m_3)^2)}. \end{aligned} \quad (10)$$

The formula for  $s'_{23}$  is similar to (10) with replaced by  $z \rightarrow -z$ . Thus  $A_{1^b}(s, s'_{13}) + A_{1^b}(s, s'_{23})$  must be replaced by  $2A_{1^b}(s, s'_{13})$ .  $\Lambda_J(i, k)$  is the cutoff at the large value of  $s_{ik}$ , which determines the contribution from small distances.

The construction of the approximate solution of coupled system-equations is based on the extraction of the leading singularities which are close to the region  $s_{ik} = (m_i + m_k)^2$  [31].

We consider the approximation, which corresponds to the single interaction of the all three particles (two-particle and triangle singularities) and neglecting all the weaker ones.

The functions  $\alpha_J(s, s_{ik})$  are the smooth functions of  $s_{ik}$  as compared with the singular part of the amplitude, hence it can be expanded in a series at the singular point and only the first term of this series should be employed further. As  $s_0$  it is convenient to take the middle point of physical region of the Dalitz-plot in which  $z = 0$ . In this case, we get from (10)  $s_{ik} = s_0 = \frac{s+m_1^2+m_2^2+m_3^2}{m_{12}^2+m_{13}^2+m_{23}^2}$ , where  $m_{ik} = \frac{m_i+m_k}{2}$ . We define functions  $\alpha_J(s, s_{ik})$  and  $b_J(s_{ik})$  at the point  $s_0$ . Such a choice of point  $s_0$  allows us to replace integral equations (4) by the algebraic couple equations for the state  $\Sigma_b^+$ :

$$\left\{ \begin{array}{l} \alpha_1(s, s_0) = \lambda + \frac{1}{2} \alpha_{1^b}(s, s_0) I_{11^b}(s, s_0) \frac{b_{1^b}(s_0)}{b_1(s_0)} + \frac{3}{2} \alpha_{0^b}(s, s_0) I_{10^b}(s, s_0) \frac{b_{0^b}(s_0)}{b_1(s_0)} \\ \alpha_{1^b}(s, s_0) = \lambda + \alpha_1(s, s_0) I_{1^b 1}(s, s_0) \frac{b_1(s_0)}{b_{1^b}(s_0)} \\ \quad - \frac{1}{2} \alpha_{1^b}(s, s_0) I_{1^b 1^b}(s, s_0) + \frac{3}{2} \alpha_{0^b}(s, s_0) I_{1^b 0^b}(s, s_0) \frac{b_{0^b}(s_0)}{b_{1^b}(s_0)} \\ \alpha_{0^b}(s, s_0) = \lambda + \alpha_1(s, s_0) I_{0^b 1}(s, s_0) \frac{b_1(s_0)}{b_{0^b}(s_0)} \\ \quad + \frac{1}{2} \alpha_{1^b}(s, s_0) I_{0^b 1^b}(s, s_0) \frac{b_{1^b}(s_0)}{b_{0^b}(s_0)} + \frac{1}{2} \alpha_{0^b}(s, s_0) I_{0^b 1^b}(s, s_0) . \end{array} \right. \quad (11)$$

The function  $I_{J_1 J_2}(s, s_0)$  takes into account singularity which corresponds to the simultaneous vanishing of all propagators in the triangle diagram.

$$I_{J_1 J_2}(s, s_0) = \int_{(m_i + m_k)^2}^{\Lambda_{J_1}} \frac{ds'_{ik}}{\pi} \frac{\rho_{J_1}(s'_{ik}) G_{J_1}^2(s'_{ik})}{s'_{ik} - s_{ik}} \int_{-1}^1 \frac{dz}{2} \frac{1}{1 - B_{J_2}(s_{ij})} . \quad (12)$$

The  $G_J(s_{ik})$  functions have the smooth dependence from energy  $s_{ik}$  [27], therefore we suggest them as constants. The parameters of model:  $g_J$  vertex constant and  $\lambda_J$  cutoff parameter are chosen dimensionless;

$$g_J = \frac{m_i + m_k}{2\pi} G_J, \quad \lambda_J = \frac{4\Lambda_J}{(m_i + m_k)^2} . \quad (13)$$

Here  $m_i$  and  $m_k$  are quark masses in the intermediate state of the quark loop. We calculate the coupled system of equations and can determine the mass values of the  $\Sigma_b^+$  state. We calculate a pole in  $s$  which corresponds to the bound state of the three quarks.

By analogy with  $\Sigma_b^+$ -hyperon we obtain the systems of equations for the reduced amplitudes of all bottom baryons. The solutions of the coupled system of equations are considered as:

$$\alpha_J = \frac{F_J(s, \lambda_J)}{D(s)} , \quad (14)$$

where the zeros of the  $D(s)$  determinate the masses of bound states of baryons.  $F_J(s, \lambda_J)$  are the functions of  $s$  and  $\lambda_J$ . The functions  $F_J(s, \lambda_J)$  determine the contributions of subamplitudes to the baryon amplitude.

In quark models, which describe rather well the masses and static properties of hadrons, the masses of the quarks usually have the similar values for the spectra of light and heavy baryons. However, this is achieved at the expense of some difference in the characteristics of the confinement potential. It should be borne in mind that for a fixed hadron mass the masses of the dressed quarks which enter into the composition of the hadron will become smaller when the slope of the confinement potential increases or its radius decreases. Therefore, conversely, we can change the masses of the dressed quarks when going from the spectrum of light baryons to the heavy baryons, while keeping the characteristics of the confinement potential unchanged. We can effectively take into account the contribution of the confinement potential in obtaining the spectrum of  $S$ -wave heavy baryons.

In the case of  $b$  quark we have used two new parameters: the cutoff of the  $bb$  diquark  $\lambda_b = 5.4$  and the coupling constant  $g_b = 1.03$ . These values have been determined by the  $b$ -baryon masses:  $M_{\Sigma_b \frac{1}{2}^+} = 5.808 \text{ GeV}$  and  $M_{\Sigma_b \frac{3}{2}^+} = 5.829 \text{ GeV}$ . In order to fix  $m_b = 4.840 \text{ GeV}$  we use the  $b$ -baryon mass  $5.829 \text{ GeV}$ . We represent the masses of all  $S$ -wave bottom baryons in the Tables I, II. The calculated mass value  $M_{\Lambda_b \frac{1}{2}^+} = 5.624 \text{ GeV}$  is equal to the experimental data [32], the mass value  $M_{\Xi_b^A \frac{1}{2}^+} = 5.761 \text{ GeV}$  is close to the experimental one. We neglect with the mass distinction of  $u$  and  $d$  quarks. The estimation of the theoretical error on the bottom baryon masses is  $2 - 5 \text{ MeV}$ . This result was obtained by the choice of model parameters.

In our model the spin-averaged mass of the states  $\Xi'_b$  and  $\Xi_b^*$  is predicted to lie around to  $250 \text{ MeV}$  above  $M_{\Xi_b}$ . The relativistic corrections are particularly important for the splitting between  $\Omega_b^+$  and  $\Omega_b$  baryons.

In the context of  $\Xi'_b$  and  $\Xi_b^*$  masses, it is worth mentioning two relations among bottom baryons which incorporate the effects of  $SU(3)_f$  breaking:

$$(M(\Sigma_b^*) - M(\Sigma_b)) + (M(\Omega_b^*) - M(\Omega_b)) - 2(M(\Xi_b^*) - M(\Xi_b')) = 0, \quad (15)$$

$$M(\Sigma_b) + M(\Omega_b) - 2M(\Xi_b') = 0. \quad (16)$$

The sign in our prediction is

$$M(\Sigma_b^*) - M(\Sigma_b) < M(\Omega_b^*) - M(\Omega_b). \quad (17)$$

This inequality is not predicted by other recent approaches [33, 34], which predict a  $\Omega_b$  splitting smaller than a  $\Sigma_b$  splitting. This suggests that the sign of the  $SU(3)$  symmetry breaking gives information about the form of the confinement potential.

We have used the  $m_b/m_c = 2.95$  in the Tables I and II (similar to the Ref. [35]).

### III. SIX-QUARK AMPLITUDES OF THE HEXAQUARKS.

We derive the relativistic six-quark equations in the framework of the dispersion relation technique. We use only planar diagrams; the other diagrams due to the rules of  $1/N_c$  expansion [24–26] are neglected. The current generates a six-quark system. The correct equations for the amplitude are obtained by taking into account all possible subamplitudes. Then one should represent a six-particle amplitude as a sum of 15 subamplitudes:

$$A = \sum_{\substack{i < j \\ i, j=1}}^6 A_{ij}. \quad (18)$$

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams. We need to consider only one group of diagrams and the amplitude corresponding to them, for example  $A_{12}$ . We shall consider the derivation of the relativistic generalization of the Faddeev-Yakubovsky approach. In our case, the low-lying dibaryons are considered. We take into account the pairwise interaction of all six quarks in the hexaquark.

For instance, we consider the state  $\Delta\Lambda_b$  with  $I = \frac{3}{2}$ ,  $J^P = 2^+$  and quark content  $uuuudb$ . The set of diagrams associated with the amplitude  $A_{12}$  can further be broken down into eight groups corresponding to subamplitudes:  $A_1^{1uu}$ ,  $A_1^{0ud}$ ,  $A_1^{0ub}$ ,  $A_1^{0db}$ ,  $A_2^{1uu0ud}$ ,  $A_2^{1uu0ub}$ ,  $A_2^{1uu0db}$ ,  $A_3^{1uu1uu0db}$ .

The amplitude  $A_1^{1uu}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$  consists of the three color sub-structures: the diquark  $1^{uu}$  in the color state  $\bar{3}_c$ , the quarks 3, 4 in the color state  $3_c \times 3_c = \bar{3}_c + 6_c$ , and the quarks 5, 6 in the color state  $3_c \times 3_c = \bar{3}_c + 6_c$ . Then we consider the total color singlet:  $\bar{3}_c \times \bar{3}_c \times \bar{3}_c = 1_c + 8_c + 8_c + 10_c^*$ . The dibaryon amplitude  $A_2^{1uu0ud}(s, s_{12345}, s_{1234}, s_{12}, s_{34})$  contains the following sub-structures: the two diquark  $1^{uu}$  and  $0^{ud}$  in the color state  $\bar{3}_c$  and the two quarks in the color state  $3_c$ . Then the dibaryon amplitude is the total color singlet. The amplitude  $A_3^{1uu1uu0db}(s, s_{12345}, s_{12}, s_{34}, s_{56})$  consists of the three diquark structures in the color state  $\bar{3}_c$ . Therefore the total color singlet can be constructed. For the others amplitudes color singlet also can be found.

The system of graphical equations (see for example equation for the amplitude  $A_2^{1uu0ud}$  for the state  $\Delta\Lambda_b$  with  $I = \frac{3}{2}$  and  $J^P = 2^+$  at the Fig. 1) is determined by the subamplitudes using the self-consistent method. The coefficients are determined by the permutation of quarks.

In order to represent the subamplitudes  $A_1^{1uu}$ ,  $A_1^{0ud}$ ,  $A_1^{0ub}$ ,  $A_1^{0db}$ ,  $A_2^{1uu0ud}$ ,  $A_2^{1uu0ub}$ ,  $A_2^{1uu0db}$ ,  $A_3^{1uu1uu0db}$  in the form of a dispersion relation, it is necessary to define the amplitude of  $qq$  and  $qQ$  interactions. We use the results of our relativistic quark model [29] and write down the pair quark amplitudes in the form:

$$a_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})}, \quad (19)$$

$$B_n(s_{ik}) = \int_{(m_i+m_k)^2}^{(m_i+m_k)^2 + \Lambda} \frac{ds'_{ik}}{\pi} \frac{\rho_n(s'_{ik}) G_n^2(s'_{ik})}{s'_{ik} - s_{ik}}, \quad (20)$$

$$\rho_n(s_{ik}, J^{PC}) = \left( \alpha(n, J^{PC}) \frac{s_{ik}}{(m_i + m_k)^2} + \beta(n, J^{PC}) + \delta(n, J^{PC}) \frac{(m_i - m_k)^2}{s_{ik}} \right) \times \frac{\sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)}}{s_{ik}}. \quad (21)$$

The coefficients  $\alpha(n, J^{PC})$ ,  $\beta(n, J^{PC})$  and  $\delta(n, J^{PC})$  are given in Table VIII. Here  $n = 1$  corresponds to  $qq$  and  $qQ$ -pairs with  $J^P = 0^+$ ,  $n = 2$  corresponds to  $qq$  and  $qQ$ -pairs with  $J^P = 1^+$ .

The coupled integral equations correspond to Fig. 1 can be described similar to [36]. Then we can go from the integration of the cosine of the angles  $dz_i$  to the integration over the subenergies.

Let us extract two- and three-particle singularities in the amplitudes  $A_1^{1^{uu}}$ ,  $A_1^{0^{ud}}$ ,  $A_1^{0^{ub}}$ ,  $A_1^{0^{db}}$ ,  $A_2^{1^{uu}0^{ud}}$ ,  $A_2^{1^{uu}0^{ub}}$ ,  $A_2^{1^{uu}0^{db}}$ ,  $A_3^{1^{uu}1^{uu}0^{db}}$ :

$$A_1^{1^{uu}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1^{uu}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) B_{1^{uu}}(s_{12})}{[1 - B_{1^{uu}}(s_{12})]}, \quad (22)$$

$$A_1^{0^{ud}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{0^{ud}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) B_{0^{ud}}(s_{12})}{[1 - B_{0^{ud}}(s_{12})]}, \quad (23)$$

$$A_1^{0^{ub}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{0^{ub}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) B_{0^{ub}}(s_{12})}{[1 - B_{0^{ub}}(s_{12})]}, \quad (24)$$

$$A_1^{0^{db}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{0^{db}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) B_{0^{db}}(s_{12})}{[1 - B_{0^{db}}(s_{12})]}, \quad (25)$$

$$A_2^{1^{uu}0^{ud}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) = \frac{\alpha_2^{1^{uu}0^{ud}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) B_{1^{uu}}(s_{12}) B_{0^{ud}}(s_{34})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{0^{ud}}(s_{34})]}, \quad (26)$$

$$A_2^{1^{uu}0^{ub}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) = \frac{\alpha_2^{1^{uu}0^{ub}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) B_{1^{uu}}(s_{12}) B_{0^{ub}}(s_{34})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{0^{ub}}(s_{34})]}, \quad (27)$$

$$A_2^{1^{uu}0^{db}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) = \frac{\alpha_2^{1^{uu}0^{db}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) B_{1^{uu}}(s_{12}) B_{0^{db}}(s_{34})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{0^{db}}(s_{34})]}, \quad (28)$$

$$A_3^{1^{uu}1^{uu}0^{db}}(s, s_{12345}, s_{12}, s_{34}, s_{56}) = \frac{\alpha_3^{1^{uu}1^{uu}0^{db}}(s, s_{12345}, s_{12}, s_{34}, s_{56}) B_{1^{uu}}(s_{12}) B_{1^{uu}}(s_{34}) B_{0^{db}}(s_{56})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{1^{uu}}(s_{34})][1 - B_{0^{db}}(s_{56})]}. \quad (29)$$

We used the classification of singularities, which was proposed in paper [31]. Using this classification, one defines the reduced amplitudes  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  as well as the  $B$ -functions in the middle point of physical region of Dalitz-plot at the point  $s_0$ .

Such choice of point  $s_0$  allows us to replace integral equations ( $\Delta\Lambda_b$ ,  $I = \frac{3}{2}$ ,  $J^P = 2^+$ ) by the algebraic equations (30) – (37):

$$\begin{aligned} \alpha_1^{1^{uu}} &= \lambda + 4\alpha_1^{1^{uu}} I_1(1^{uu}1^{uu}) + 2\alpha_1^{0^{ud}} I_1(1^{uu}0^{ud}) + 2\alpha_1^{0^{ub}} I_1(1^{uu}0^{ub}) + 4\alpha_2^{1^{uu}0^{ud}} I_2(1^{uu}1^{uu}0^{ud}) \\ &+ 4\alpha_2^{1^{uu}0^{ub}} I_2(1^{uu}1^{uu}0^{ub}), \end{aligned} \quad (30)$$

$$\begin{aligned} \alpha_1^{0^{ud}} &= \lambda + 3\alpha_1^{1^{uu}} I_1(0^{ud}1^{uu}) + 3\alpha_1^{0^{ud}} I_1(0^{ud}0^{ud}) + \alpha_1^{0^{ub}} I_1(0^{ud}0^{ub}) + \alpha_1^{0^{db}} I_1(0^{ud}0^{db}) \\ &+ 6\alpha_2^{1^{uu}0^{ud}} I_2(0^{ud}1^{uu}0^{ud}) + 3\alpha_2^{1^{uu}0^{db}} I_2(0^{ud}1^{uu}0^{db}), \end{aligned} \quad (31)$$

$$\begin{aligned}
\alpha_1^{0^{ub}} &= \lambda + 3\alpha_1^{1^{uu}} I_1(0^{ub}1^{uu}) + \alpha_1^{0^{ud}} I_1(0^{ub}0^{ud}) + 3\alpha_1^{0^{ub}} I_1(0^{ub}0^{ub}) + \alpha_1^{0^{db}} I_1(0^{ub}0^{db}) \\
&+ 6\alpha_2^{1^{uu}0^{ub}} I_2(0^{ub}1^{uu}0^{ub}) + 3\alpha_2^{1^{uu}0^{db}} I_2(0^{ub}1^{uu}0^{db}), \tag{32}
\end{aligned}$$

$$\alpha_1^{0^{db}} = \lambda + 4\alpha_1^{0^{ud}} I_1(0^{db}0^{ud}) + 4\alpha_1^{0^{ub}} I_1(0^{db}0^{ub}), \tag{33}$$

$$\begin{aligned}
\alpha_2^{1^{uu}0^{ud}} &= \lambda + \alpha_1^{1^{uu}} (2I_3(1^{uu}0^{ud}1^{uu}) + 2I_4(1^{uu}0^{ud}1^{uu})) + 2\alpha_1^{0^{ud}} I_3(1^{uu}0^{ud}0^{ud}) + \alpha_1^{0^{ub}} I_4(0^{ud}1^{uu}0^{ub}) \\
&+ \alpha_1^{0^{db}} I_4(0^{ud}1^{uu}0^{db}) + 2\alpha_2^{1^{uu}0^{ud}} I_7(0^{ud}1^{uu}0^{ud}1^{uu}) + \alpha_2^{1^{uu}0^{ub}} (2I_5(1^{uu}0^{ud}1^{uu}0^{ub}) \\
&+ 2I_6(1^{uu}0^{ud}1^{uu}0^{ub})) + \alpha_2^{1^{uu}0^{db}} (I_5(0^{ud}1^{uu}1^{uu}0^{db}) + 2I_6(1^{uu}0^{ud}1^{uu}0^{db}) + 2I_7(1^{uu}0^{ud}1^{uu}0^{db})) \\
&+ 2\alpha_3^{1^{uu}1^{uu}0^{db}} I_8(1^{uu}0^{ud}1^{uu}1^{uu}0^{db}), \tag{34}
\end{aligned}$$

$$\begin{aligned}
\alpha_2^{1^{uu}0^{ub}} &= \lambda + \alpha_1^{1^{uu}} (2I_3(1^{uu}0^{ub}1^{uu}) + 2I_4(1^{uu}0^{ub}1^{uu})) + \alpha_1^{0^{ud}} I_4(0^{ub}1^{uu}0^{ud}) + 2\alpha_1^{0^{ub}} I_3(1^{uu}0^{ub}0^{ub}) \\
&+ \alpha_1^{0^{db}} I_4(0^{ub}1^{uu}0^{db}) + \alpha_2^{1^{uu}0^{ud}} (2I_5(1^{uu}0^{ub}1^{uu}0^{ud}) + 2I_6(1^{uu}0^{ub}1^{uu}0^{ud})) \\
&+ 2\alpha_2^{1^{uu}0^{ub}} I_7(0^{ub}1^{uu}0^{ub}1^{uu}) + \alpha_2^{1^{uu}0^{db}} (I_5(0^{ub}1^{uu}1^{uu}0^{db}) + 2I_6(1^{uu}0^{ub}1^{uu}0^{db}) \\
&+ 2I_7(1^{uu}0^{ub}1^{uu}0^{db})) + 2\alpha_3^{1^{uu}1^{uu}0^{db}} I_8(1^{uu}0^{ub}1^{uu}1^{uu}0^{db}), \tag{35}
\end{aligned}$$

$$\begin{aligned}
\alpha_2^{1^{uu}0^{db}} &= \lambda + 4\alpha_1^{1^{uu}} I_4(1^{uu}0^{db}1^{uu}) + \alpha_1^{0^{ud}} (2I_3(1^{uu}0^{db}0^{ud}) + 2I_4(0^{db}1^{uu}0^{ud})) + \alpha_1^{0^{ub}} (2I_3(1^{uu}0^{db}0^{ub}) \\
&+ 2I_4(0^{db}1^{uu}0^{ub})) + \alpha_2^{1^{uu}0^{ud}} (4I_6(1^{uu}0^{db}1^{uu}0^{ud}) + 4I_7(0^{db}1^{uu}0^{ud}1^{uu})) \\
&+ \alpha_2^{1^{uu}0^{ub}} (4I_6(1^{uu}0^{db}1^{uu}0^{ub}) + 4I_7(0^{db}1^{uu}0^{ub}1^{uu})), \tag{36}
\end{aligned}$$

$$\begin{aligned}
\alpha_3^{1^{uu}1^{uu}0^{db}} &= \lambda + 4\alpha_1^{1^{uu}} I_9(1^{uu}1^{uu}0^{db}1^{uu}) + 4\alpha_1^{0^{ud}} I_9(1^{uu}0^{db}1^{uu}0^{ud}) + 4\alpha_1^{0^{ub}} I_9(1^{uu}0^{db}1^{uu}0^{ub}) \\
&+ 8\alpha_2^{1^{uu}0^{ud}} I_{10}(1^{uu}1^{uu}0^{db}1^{uu}0^{ud}) + 8\alpha_2^{1^{uu}0^{ub}} I_{10}(1^{uu}1^{uu}0^{db}1^{uu}0^{ub}), \tag{37}
\end{aligned}$$

where  $\lambda_i$  are the current constants. We used the functions  $I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}$ :

$$\begin{aligned}
I_1(ij) &= \frac{B_j(s_0^{13})}{B_i(s_0^{12})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \int_{-1}^{+1} \frac{dz_1(1)}{2} \frac{1}{1 - B_j(s'_{13})}, \tag{38} \\
I_2(ijk) &= \frac{B_j(s_0^{13})B_k(s_0^{24})}{B_i(s_0^{12})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(2)}{2} \int_{-1}^{+1} \frac{dz_2(2)}{2} \\
&\times \int_{z_3(2)^-}^{z_3(2)^+} dz_3(2) \frac{1}{\sqrt{1 - z_1^2(2) - z_2^2(2) - z_3^2(2) + 2z_1(2)z_2(2)z_3(2)}} \\
&\times \frac{1}{1 - B_j(s'_{13})} \frac{1}{1 - B_k(s'_{24})}, \tag{39}
\end{aligned}$$



$$\begin{aligned}
I_3(ijk) &= \frac{B_k(s_0^{23})}{B_i(s_0^{12})B_j(s_0^{34})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\
&\times \int_{(m_3+m_4)^2}^{\frac{(m_3+m_4)^2\Lambda_j}{4}} \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34})\rho_j(s'_{34})}{s'_{34} - s_0^{34}} \int_{-1}^{+1} \frac{dz_1(3)}{2} \int_{-1}^{+1} \frac{dz_2(3)}{2} \frac{1}{1 - B_k(s'_{23})}, \quad (40)
\end{aligned}$$

$$I_4(ijk) = I_1(ik), \quad (41)$$

$$I_5(ijkl) = I_2(ikl), \quad (42)$$

$$I_6(ijkl) = I_1(ik) \cdot I_1(jl), \quad (43)$$

$$\begin{aligned}
I_7(ijkl) &= \frac{B_k(s_0^{23})B_l(s_0^{45})}{B_i(s_0^{12})B_j(s_0^{34})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\
&\times \int_{(m_3+m_4)^2}^{\frac{(m_3+m_4)^2\Lambda_j}{4}} \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34})\rho_j(s'_{34})}{s'_{34} - s_0^{34}} \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(7)}{2} \int_{-1}^{+1} \frac{dz_2(7)}{2} \int_{-1}^{+1} \frac{dz_3(7)}{2} \\
&\times \int_{z_4(7)^-}^{z_4(7)^+} dz_4(7) \frac{1}{\sqrt{1 - z_1^2(7) - z_3^2(7) - z_4^2(7) + 2z_1(7)z_3(7)z_4(7)}} \\
&\times \frac{1}{1 - B_k(s'_{23})} \frac{1}{1 - B_l(s'_{45})}, \quad (44)
\end{aligned}$$

$$\begin{aligned}
I_8(ijklm) &= \frac{B_k(s_0^{15})B_l(s_0^{23})B_m(s_0^{46})}{B_i(s_0^{12})B_j(s_0^{34})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\
&\times \int_{(m_3+m_4)^2}^{\frac{(m_3+m_4)^2\Lambda_j}{4}} \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34})\rho_j(s'_{34})}{s'_{34} - s_0^{34}} \\
&\times \frac{1}{(2\pi)^2} \int_{-1}^{+1} \frac{dz_1(8)}{2} \int_{-1}^{+1} \frac{dz_2(8)}{2} \int_{-1}^{+1} \frac{dz_3(8)}{2} \int_{z_4(8)^-}^{z_4(8)^+} dz_4(8) \int_{-1}^{+1} \frac{dz_5(8)}{2} \int_{z_6(8)^-}^{z_6(8)^+} dz_6(8) \\
&\times \frac{1}{\sqrt{1 - z_1^2(8) - z_3^2(8) - z_4^2(8) + 2z_1(8)z_3(8)z_4(8)}} \\
&\times \frac{1}{\sqrt{1 - z_2^2(8) - z_5^2(8) - z_6^2(8) + 2z_2(8)z_5(8)z_6(8)}} \\
&\times \frac{1}{1 - B_k(s'_{15})} \frac{1}{1 - B_l(s'_{23})} \frac{1}{1 - B_m(s'_{46})}, \quad (45)
\end{aligned}$$

$$I_9(ijkl) = I_3(ijl), \quad (46)$$

$$\begin{aligned}
I_{10}(ijklm) = & \frac{B_l(s_0^{23})B_m(s_0^{45})}{B_i(s_0^{12})B_j(s_0^{34})B_k(s_0^{56})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\
& \times \int_{(m_3+m_4)^2}^{\frac{(m_3+m_4)^2\Lambda_j}{4}} \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34})\rho_j(s'_{34})}{s'_{34} - s_0^{34}} \int_{(m_5+m_6)^2}^{\frac{(m_5+m_6)^2\Lambda_k}{4}} \frac{ds'_{56}}{\pi} \frac{G_k^2(s_0^{56})\rho_k(s'_{56})}{s'_{56} - s_0^{56}} \\
& \times \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(10)}{2} \int_{-1}^{+1} \frac{dz_2(10)}{2} \int_{-1}^{+1} \frac{dz_3(10)}{2} \int_{-1}^{+1} \frac{dz_4(10)}{2} \int_{z_5(10)^-}^{z_5(10)^+} dz_5(10) \\
& \times \frac{1}{\sqrt{1 - z_1^2(10) - z_4^2(10) - z_5^2(10) + 2z_1(10)z_4(10)z_5(10)}} \\
& \times \frac{1}{1 - B_l(s'_{23})} \frac{1}{1 - B_m(s'_{45})}, \tag{47}
\end{aligned}$$

where  $i, j, k, l, m$  correspond to the diquarks with the spin-parity  $J^P = 0^+, 1^+$ .

The solutions of the system of equations are considered as:

$$\alpha_i(s) = F_i(s, \lambda_i)/D(s), \tag{48}$$

where zeros of  $D(s)$  determinants define the masses of bound states of dibaryons.

#### IV. CALCULATION RESULTS.

The model in question has three parameters of previous model [36]: gluon coupling constants  $g_0 = 0.653$  (diquark  $0^+$ ) and  $g_1 = 0.292$  (diquark  $1^+$ ), cutoff parameter  $\Lambda = 11$ . We used to the cutoff  $\Lambda_{qb} = 4.43$  and the cutoff  $\Lambda_{qc} = 5.18$ , which are determined by  $M = 7300 \text{ MeV}$  (threshold  $7315 \text{ MeV}$ ) and  $M = 4100 \text{ MeV}$  (threshold  $4130 \text{ MeV}$ ). The experimental data is absent, therefore we use the dimensionless parameters, which are similar to the Eq. (13). It allows us to calculate the mass spectra of  $qqqqqQ$  states.

The quark masses of the model are  $m_q = 495 \text{ MeV}$ ,  $m_c = 1655 \text{ MeV}$ ,  $m_b = 4840 \text{ MeV}$ . The estimation of theoretical error on the  $S$ -wave hexaquarks masses is  $1 \text{ MeV}$ . This results was obtained by the choice of model parameters.

We have considered 38 dibaryons with content  $qqqqqQ$ ,  $q = u, d$ ,  $Q = c, b$ . The masses of dibaryons with  $I = \frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{5}{2}$  and spin-parity  $J^P = 0^+, 1^+, 2^+$  in the Tables IV and V are given. The lowest mass for the  $qqqqqb$  states is  $M = 5700 \text{ MeV}$ . The lowest mass for the  $qqqqqc$  states is  $M = 3475 \text{ MeV}$ .

The relativistic six-body approach possesses the dynamical mixing and allows us to calculate the contributions of the subamplitudes to the hexaquark amplitude (Tables VI, VII). The calculated dibaryon subamplitudes  $A_2$  present the main contributions to the hexaquark amplitude (about 80 percents).

In a strongly bound systems, which include the light quarks, where  $p/m \sim 1$ , the approximation by nonrelativistic kinematics and dynamics is not justified.

In our paper, the relativistic description of three-particles amplitudes of bottom baryons are considered. We take into account the  $u, d, c, b$  quarks. The mass spectrum of  $S$ -wave bottom baryons with one, two, and three  $b$  quarks is considered. We use only two new parameters for the calculation of 23 baryon masses. The other model parameters in the our papers [21–23] are given.

We have predicted the masses of baryons containing  $b$  quarks using the coupled-channel formalism. We believe that the prediction for the  $S$ -wave bottom baryons based on the relativistic kinematics and dynamics allows as to take into account the relativistic corrections. In our consideration, the bottom baryon masses are heavier than the masses in the other quark models [37, 38].

Our model is confined to the quark-antiquark pair production on account of the phase space restriction. Here  $m_q$  is the "mass" of the constituent quark. Therefore the production of new quark-antiquark pair is absent for the low-lying hadrons.

We manage with quark as with real particles. However, in the soft region, the quark diagrams should be treated as spectral integral over quark masses with the spectral density  $\rho(m^2)$ : the integration over quark masses in the

amplitudes puts away the quark singularities and introduces the hadron ones. One can believe that the approximation:  $\rho(m^2) \rightarrow \delta(m^2 - m_q^2)$  could be possible for the low-lying hadrons.

The authors of Ref. [38] calculated the energies of the baryon-baryon threshold as a function of the flavor-symmetry breaking parameter  $\delta = 1 - \frac{m_u}{m_s}$ . The binding energy is obtained and the possible candidates for stability under strong interactions is commented on. In the recent papers [39, 40], developed for the baryon-baryon interactions in lattice QCD, the flavor-singlet H dibaryon is studied. The results of the lattice QCD calculations presented the first clear evidence for a bound state of two baryon directly from QCD.

### Acknowledgments

S.M. Gerasyuta is indebted to T. Barnes, C.-Y. Wong for useful discussions. This research was supported in part by the Russian Ministry of Education under Grant 2.1.1.68.26.

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TABLE I: Bottom baryon masses of multiplet  $\frac{1}{2}^+$ . Parameters of model: quark masses  $m_{u,d} = 495 \text{ MeV}$ ,  $m_s = 770 \text{ MeV}$ ,  $m_c = 1655 \text{ MeV}$ ,  $m_b = 4840 \text{ MeV}$ ; cutoff parameters:  $\lambda_q = 10.7$  ( $q = u, d, s$ ),  $\lambda_c = 6.5$ ,  $\lambda_b = 5.4$ ; gluon coupling constants:  $g_0 = 0.70$ ,  $g_1 = 0.55$  with  $J^p = 0^+$  and  $1^+$ ,  $g_c = 0.857$ ,  $g_b = 1.03$ .

Baryon	Mass (GeV)	Mass (GeV) (exp.)
$\Sigma_b$	5.808	5.808
$\Lambda_b$	5.624	5.624
$\Xi_{sb}^A$	5.761	5.774, 5.793
$\Xi_{sb}^S$	6.007	—
$\Omega_{ssb}$	6.120	—
$\Xi_{cb}^A$	6.789	—
$\Xi_{cb}^S$	6.818	—
$\Lambda_{scb}^A$	6.798	—
$\Lambda_{scb}^S$	6.836	—
$\Omega_{ccb}$	7.943	—
$\Xi_{bb}$	10.045	—
$\Omega_{sbb}$	9.999	—
$\Omega_{cbb}$	11.089	—

TABLE II: Bottom baryon masses of multiplet  $\frac{3}{2}^+$ . Parameters of model: quark masses  $m_{u,d} = 495 \text{ MeV}$ ,  $m_s = 770 \text{ MeV}$ ,  $m_c = 1655 \text{ MeV}$ ,  $m_b = 4840 \text{ MeV}$ ; cutoff parameters:  $\lambda_q = 10.7$  ( $q = u, d, s$ ),  $\lambda_c = 6.5$ ,  $\lambda_b = 5.4$ ; gluon coupling constants:  $g_0 = 0.70$ ,  $g_1 = 0.55$  with  $J^p = 0^+$  and  $1^+$ ,  $g_c = 0.857$ ,  $g_b = 1.03$ .

Baryon	Mass (GeV)	Mass (GeV) (exp.)
$\Sigma_b$	5.829	5.829
$\Xi_{sb}$	6.066	—
$\Omega_{ssb}$	6.220	—
$\Xi_{cb}$	6.863	—
$\Omega_{scb}$	6.914	—
$\Omega_{ccb}$	7.973	—
$\Xi_{bb}$	10.104	—
$\Omega_{sbb}$	10.126	—
$\Omega_{cbb}$	11.123	—
$\Omega_{bbb}$	14.197	—

TABLE III: Coefficients of Ghew-Mandelstam functions.

	$\alpha_J$	$\beta_J$	$\delta_J$
$1^+$	$\frac{1}{3}$	$\frac{4m_i m_k}{3(m_i + m_k)^2} - \frac{1}{6}$	$-\frac{1}{6}(m_i - m_k)^2$
$0^+$	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_i - m_k)^2}{(m_i + m_k)^2}$	0

TABLE IV: S-wave charmed dibaryon masses. Parameters of model: cutoff  $\Lambda = 11.0$  and  $\Lambda_{qc} = 5.18$ , gluon coupling constants  $g_0 = 0.653$  and  $g_1 = 0.292$ . Quark masses  $m_q = 495 \text{ MeV}$  and  $m_c = 1655 \text{ MeV}$ .

$I$	$J$	Dibaryons (quark content)	Mass (MeV)
$\frac{5}{2}$	0	$\Delta\Sigma_c^* (uuu \ uuc)$	4100
	1, 2	$\Delta\Sigma_c, \Delta\Sigma_c^* (uuu \ uuc)$	4100
$\frac{3}{2}$	0	$\Delta\Sigma_c^* (uuu \ udc + uud \ uuc)$	3570
		$N\Sigma_c (uud \ uuc)$	3699
	1	$\Delta\Sigma_c, \Delta\Sigma_c^* (uuu \ udc + uud \ uuc)$	3570
		$\Delta\Lambda_c (uuu \ udc)$	3920
		$N\Sigma_c, N\Sigma_c^* (uud \ uuc)$	3699
	2	$\Delta\Sigma_c, \Delta\Sigma_c^* (uuu \ udc + uud \ uuc)$	3746
		$\Delta\Lambda_c (uuu \ udc)$	3920
		$N\Sigma_c^* (uud \ uuc)$	3902
$\frac{1}{2}$	0	$\Delta\Sigma_c^* (uuu \ ddc + uud \ udc + udd \ uuc)$	3475
		$N\Lambda_c (uud \ udc)$	3629
		$N\Sigma_c (uud \ udc + udd \ uuc)$	3480
	1	$\Delta\Sigma_c, \Delta\Sigma_c^* (uuu \ ddc + uud \ udc + udd \ uuc)$	3475
		$N\Lambda_c, \Delta\Lambda_c (uud \ udc)$	3629
		$N\Sigma_c, N\Sigma_c^* (uud \ udc + udd \ uuc)$	3480
	2	$\Delta\Sigma_c, \Delta\Sigma_c^* (uuu \ ddc + uud \ udc + udd \ uuc)$	3863
		$\Delta\Lambda_c (uud \ udc)$	3937
		$N\Sigma_c^* (uud \ udc + udd \ uuc)$	3870

TABLE V: S-wave bottom dibaryon masses. Parameters of model: cutoff  $\Lambda = 11.0$  and  $\Lambda_{qb} = 4.43$ , gluon coupling constants  $g_0 = 0.653$  and  $g_1 = 0.292$ . Quark masses  $m_q = 495 \text{ MeV}$  and  $m_b = 4840 \text{ MeV}$ .

$I$	$J$	Dibaryons (quark content)	Mass (MeV)	
$\frac{5}{2}$	0	$\Delta\Sigma_b^* (uuu uub)$	7300	
	1, 2	$\Delta\Sigma_b, \Delta\Sigma_b^* (uuu uub)$	7300	
$\frac{3}{2}$	0	$\Delta\Sigma_b^* (uuu udb + uud uub)$	5988	
		$N\Sigma_b (uud uub)$	6285	
	1	$\Delta\Sigma_b, \Delta\Sigma_b^* (uuu udb + uud uub)$	5988	
		$\Delta\Lambda_b (uuu udb)$	6926	
		$N\Sigma_b, N\Sigma_b^* (uud uub)$	6285	
	2	$\Delta\Sigma_b, \Delta\Sigma_b^* (uuu udb + uud uub)$	6450	
		$\Delta\Lambda_b (uuu udb)$	6926	
		$N\Sigma_b^* (uud uub)$	6800	
	$\frac{1}{2}$	0	$\Delta\Sigma_b^* (uuu ddb + uud udb + udd uub)$	5700
			$N\Lambda_b (uud udb)$	6142
		$N\Sigma_b (uud udb + udd uub)$	5723	
1		$\Delta\Sigma_b, \Delta\Sigma_b^* (uuu ddb + uud udb + udd uub)$	5700	
		$N\Lambda_b, \Delta\Lambda_b (uud udb)$	6142	
		$N\Sigma_b, N\Sigma_b^* (uud udb + udd uub)$	5723	
2		$\Delta\Sigma_b, \Delta\Sigma_b^* (uuu ddb + uud udb + udd uub)$	6744	
		$\Delta\Lambda_b (uud udb)$	6928	
		$N\Sigma_b^* (uud udb + udd uub)$	6755	

TABLE VI:  $\Delta\Sigma_c, \Delta\Sigma_c^* (3475 \text{ MeV})$  ( $IJ = \frac{1}{2}1$ ). Parameters of model: cutoff  $\Lambda = 11.0$  and  $\Lambda_{qc} = 5.18$ , gluon coupling constants  $g_0 = 0.653$  and  $g_1 = 0.292$ . Quark masses  $m_q = 495 \text{ MeV}$  and  $m_c = 1655 \text{ MeV}$ .

Subamplitudes	Contributions, percent
$A_1^{uu}$	4.98
$A_1^{dd}$	4.20
$A_0^{ud}$	3.81
$A_1^{uc}$	0.82
$A_0^{dc}$	0.23
$A_2^{uu1dd}$	9.58
$A_2^{uu0ud}$	17.73
$A_2^{uu0uc}$	1.17
$A_2^{uu0dc}$	1.69
$A_2^{dd0uc}$	1.43
$A_2^{ud0ud}$	46.54
$A_2^{ud0uc}$	3.23
$A_2^{ud0dc}$	3.11
$A_3^{uu1dd0uc}$	0.43
$A_3^{uu0ud0dc}$	1.06
$\sum A_1$	14.05
$\sum A_2$	84.46
$\sum A_3$	1.49

TABLE VII:  $\Delta\Sigma_b$ ,  $\Delta\Sigma_b^*$  (5700 MeV) ( $IJ = \frac{1}{2}1$ ). Parameters of model: cutoff  $\Lambda = 11.0$  and  $\Lambda_{qb} = 4.43$ , gluon coupling constants  $g_0 = 0.653$  and  $g_1 = 0.292$ . Quark masses  $m_q = 495$  MeV and  $m_b = 4840$  MeV.

Subamplitudes	Contributions, percent
$A_1^{1^{uu}}$	5.79
$A_1^{1^{dd}}$	4.89
$A_1^{0^{ud}}$	2.67
$A_1^{0^{ub}}$	0.11
$A_1^{0^{db}}$	0.02
$A_2^{1^{uu}1^{dd}}$	11.04
$A_2^{1^{uu}0^{ud}}$	19.82
$A_2^{1^{uu}0^{ub}}$	0.13
$A_2^{1^{uu}0^{db}}$	0.21
$A_2^{1^{dd}0^{ub}}$	0.16
$A_2^{0^{ud}0^{ud}}$	54.15
$A_2^{0^{ud}0^{ub}}$	0.42
$A_2^{0^{ud}0^{db}}$	0.39
$A_3^{1^{uu}1^{dd}0^{ub}}$	0.06
$A_3^{1^{uu}0^{ud}0^{db}}$	0.14
$\sum A_1$	13.47
$\sum A_2$	86.33
$\sum A_3$	0.20

TABLE VIII: Vertex functions and Chew-Mandelstam coefficients.

$i$	$G_i^2(s_{kl})$	$\alpha_i$	$\beta_i$	$\delta_i$
$0^+$	$\frac{4g}{3} - \frac{8gm_{kl}^2}{(3s_{kl})}$	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$	0
$1^+$	$\frac{2g}{3}$	$\frac{1}{3}$	$\frac{4m_k m_l}{3(m_k + m_l)^2} - \frac{1}{6}$	$-\frac{1}{6} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$

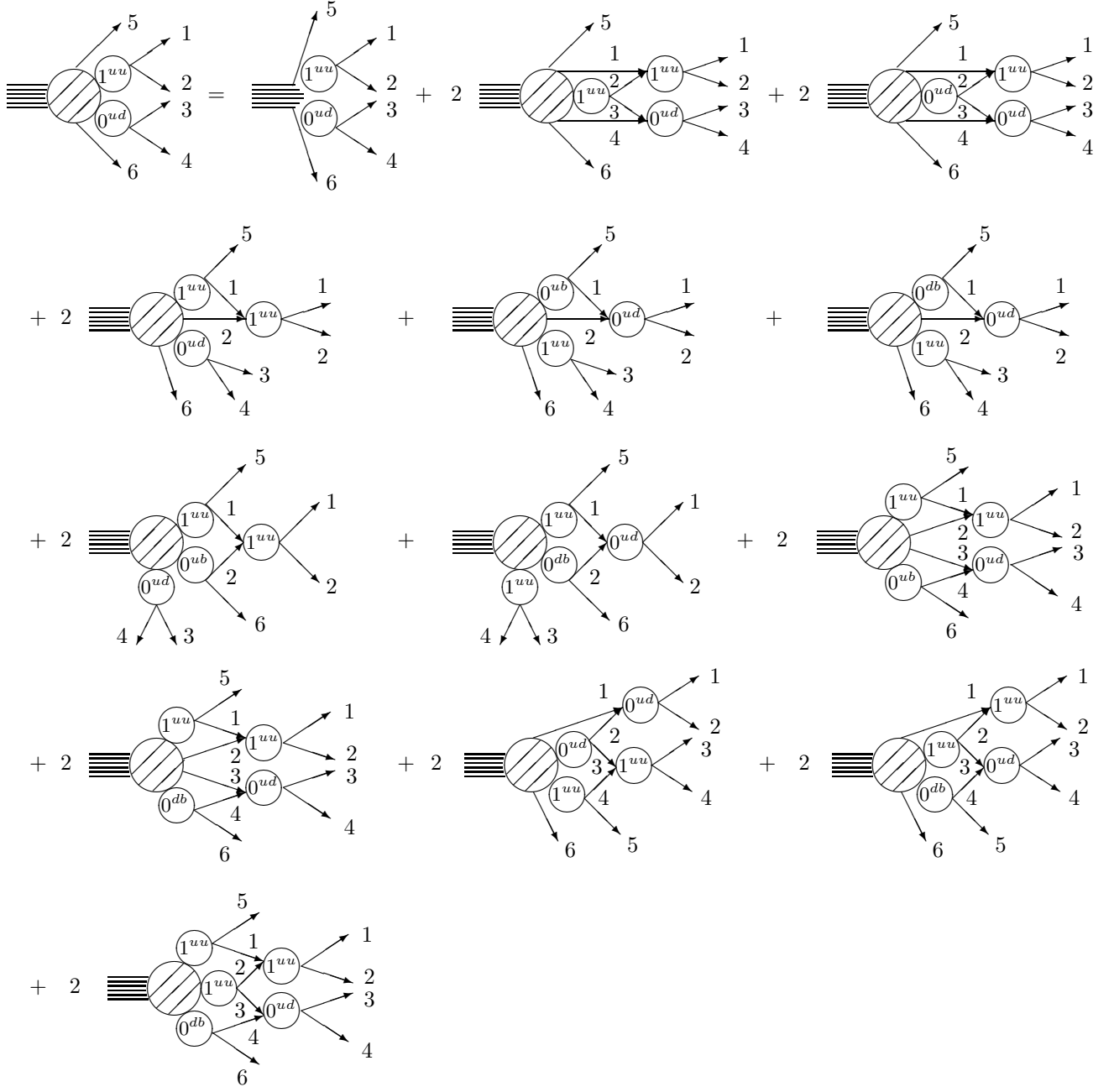


Fig. 1. Graphical representation of the amplitude  $A_2^{1^{uu}0^{ud}}$  for the case of  $\Delta\Lambda_b$ ,  $I = \frac{3}{2}$ ,  $J^P = 2^+$ .